

Fudan University

复旦大学

2012-2013 学年第二学期考试试卷

课程名称: 离散数学 (II)
开课院系: 软件学院

课程代码: SOFT130040.01
考试形式: 开卷

姓名: _____ 学号: _____ 专业: _____

题目	1	2	3	4	5	6	7	8	9	10	总分
得分											

DIRECTION: There are totally **two** pages of examination paper. You must write all your answers, include your name and student number clearly, on a specific answersheet. You have 2 hours to solve all the questions.

- Given a poset $\{1, 2, 4, 5, 10, 20\}$ with the relation defined by Figure 1,
 - Verify it is a lattice? (5 points)
 - Is $\{1, 4, 10, 20\}$ a sublattice of $\{1, 2, 4, 5, 10, 20\}$? Give your reasons. (5 points)

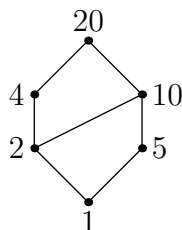


Figure 1: Lattice

- If P and Q are posets, let Q^P be the poset of order-preserving maps from P to Q , where for $f, g \in Q^P$ we define $f \leq g$ iff $f(a) \leq g(a)$ for all $a \in P$. If Q is a lattice show that Q^P is also a lattice. (5 points)
- Prove or disprove the following statements. If a statement is false, give a counterexample. (15 marks)
 - Let α be a well-defined proposition. α has arbitrary length when length $k \geq 4$. (3 marks)
 - B is satisfiable if and only if $(\neg B)$ is not valid. (3 marks)
 - If a proposition α is satisfiable, then it has infinite models. (3 marks)
 - The number of tautologies is the same as the number of unsatisfiable propositions. (3 marks)

- (e) Given two sets of propositions Σ_1 and Σ_2 , $\Sigma_1 \subseteq \Sigma_2 \Rightarrow \mathcal{M}(\Sigma_2) \subseteq \mathcal{M}(\Sigma_1)$.
4. Let F be the constant false and M be the ternary minority connectives($M(A, B, C)$ is true if and only if at most one of A, B, C is true). Show that $\{F, M\}$ is complete. (5 marks)
5. Prove that every proposition can be represented in CNF(Conjunctive Normal Form).(8 marks)
6. Prove or disprove the following statements. If it is false, a counterexample is needed:(10 marks)
- (a) $\{(\neg(\beta \rightarrow \gamma) \rightarrow \neg\alpha), \beta\} \models (\alpha \rightarrow \gamma)$.(4 marks)
- (b) $\{p \rightarrow (q \vee r), \neg q, \neg r\} \vdash \neg p$.(3 marks)
- (c) $((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R)))$. (3 marks)
7. Given the following deduction:

If the equatorial rain forests produce oxygen used by Americans, then either Americans ought to pay for the oxygen, or they ought to stop complaining about the destruction of the rain forests. But either it is false that Americans ought to pay for the oxygen, or it is false that Americans ought to stop complaining about the destruction of the rain forests. Therefore, it is false that the equatorial rain forests produce oxygen used by Americans.

Answer the following questions: (9 marks)

- (a) Formalize the deduction with propositional logic. (5 marks)
- (b) Prove or disprove the conclusion formally. (4 marks)
8. Given a set $\Sigma = \{\alpha_1, \dots, \alpha_n\}$ of finite propositions, prove that $\Sigma \models \alpha$ if and only if $(\alpha_1 \wedge \dots \wedge \alpha_n) \rightarrow \alpha$ is valid.(8 marks)
9. In 1977 it was proved that every planar map can be colored with four colors(Of course there are only finite countries on map). Suppose we have an infinite(but countable) planar map with countries C_1, C_2, C_3, \dots (10 marks)
- (a) Formalize this problem with propositional logic. (5 marks)
- (b) Prove that an infinite map can be still colored with four colors. (5 marks)