Fudan University

复旦大学

2011-2012学年第2学期考试试卷

课程名称: 离散数学(II) 课程代码: SOFT130040.01
开课院系: 软件学院 考试形式: 开卷

姓 名:_____ 学 号:_____ 专 业:_____

题目	1	2	3	4	5	6	7	8	9	10	总分
得分											

DIRECTION: There are totally **two** pages and **60** marks of examination paper. You must write all your answers, include your name and student number clearly, on a given answersheet. You have 2 hours to solve all the questions. Please mark your name and id on each answer sheet.

1. Give the following statements:

   (a) If I graduate this semester, then I will have passed the physics course.

   (b) If I do not study physics for 10 hours a week, then I would not pass physics.

   (c) If I study physics for 10 hours a week, then I can not play volleyball.

   Answer the following questions: (9 marks)

   (a) Formalize them in logic approach. (5 marks)

   (b) Prove or disprove the assertion: if I play volleyball, I will not graduate this semester. (4 marks)

2. Define a binary connective $A \oplus B$ whose truth table is given in Table **??**. Answer

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Table 1: Truth table of $\oplus$

   the following questions.(7 marks)

   (a) Is $\{\oplus\}$ adequate? (2 marks)

(b) Add the atomic tableau of $\oplus$ into atomic tableaux. (2 marks)

(c) Discuss the validity of $(A \oplus B) \to ((A \wedge \neg B) \vee (\neg A \wedge B))$ by using tableau proof. (3 marks)

3. Given a formula $\Phi(x, y) = (\varphi(x, y) \to (\forall x)(\psi(x) \vee (\exists x)\varphi(x, y)))$, answer the following questions.(8 marks)

(a) Show which occurrence of every variable is free. If it is bound, show which quantifier bounds it. (3 marks)

(b) Given a function $f(z, x)$, is $y/f(z, x)$ in $\Phi(x, y)$ substitutable? Show your reason. (2 marks)

(c) Discuss its truth of $\Phi(x, y)$. (3 marks)

4. Prove or disprove the following statements using tableau proof system. If it is false, a counterexample is needed: (9 marks)

(a) $\{A \vee \neg B, B \vee \neg C, C \vee \neg D\} \vdash A \to D$. (3 marks)

(b) $\forall x(\phi(x) \vee \psi(x)) \leftrightarrow (\forall x\phi(x) \vee \forall x\psi(x))$. (3 marks)

(c) $\forall x\exists y(\varphi \vee \exists z\psi) \to \forall x\exists y\exists w(\varphi \vee \psi(z/w))$, where φ and ψ are any formulas with free variables x, y and z. w is a variable not appearing in φ and ψ. (3 marks)

5. Give two sentence $\alpha = (\forall x P(x)) \to Q$ and $\beta = \forall x(P(x) \to Q)$. (9 marks)

(a) Prove that $\beta \models \alpha$ in semantics approach. (4 marks)

(b) Prove it by using tableau proof. (2 marks)

(c) If x is free in Q, discuss the truth of the following assertion $((\forall x P(x)) \to Q(x)) \vdash \forall x((P(x) \to Q(x))$. (3 marks)

6. Construct a set of sentences S and prove that it has only infinite models. (4 marks)

7. Let $S = \{\phi_i(x_1, \ldots, x_{n_i}) | i \leq n \text{ for some } n\}$ be a set of open formulas on top of a language $\mathcal{L}$. If S is unsatisfiable, there are only finitely many ground instance of elements of S whose conjunction is unsatisfiable. (5 marks)

8. A graph $G = \langle V, E\rangle$ is 2-*colorable* if and only if G has no odd cycle. Given a binary predicate $E(x, y)$, which means that there is a edge between vertex x and vertex y. (9 marks)

(a) Construct a sentence ϕ_n to represent there is no cycle with n-length. (*Hints*: Construct it recursively.) (3 marks)

(b) Use a set of sentences Σ to describe that G is 2-*colorable*. (2 marks)

(c) Prove that a graph is 2-*colorable* cannot be described by a sentence ψ. (*Hints*: Consider $\neg\psi$ and sentence set Σ, then apply Compactness Theorem.) (4 marks)