复旦大学物理学系

2006～2007 学年第二学期期末考试试卷

A 卷

课程名称：___大学物理（下）___ 课程代码：PHYS120002.05

开课院系：___物理学系___ 考试形式：___闭卷___

姓　名：________ 学　号：________ 专　业：________

题 号	1	2	3	4	5	6	总 分
得 分							

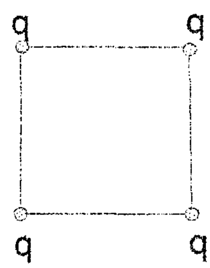
Electric constant $\varepsilon_0 = 8.85\times10^{-12} C^2/(N\cdot m^2)$

Magnetic constant $\mu_0 = 4\pi\times10^{-7} N/A^2 = 1.26\times10^{-6} H/m$

Plank constant $h = 6.63\times10^{-34} J\cdot s = 4.14\times10^{-15} eV\cdot s$

Neutron mass $m_n = 1.675\times10^{-27} kg$

1. Derive an expression for the work required by an external agent to put the four charges together as indicated in the figure below. Each side of the square has length a. (15%)



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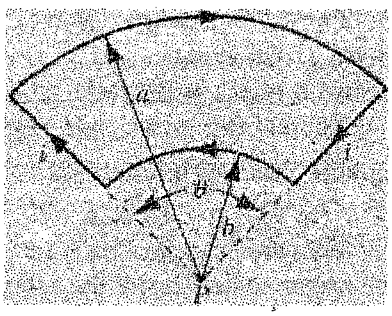
2. In the laboratory we produce magnetic fields using current-carrying wires rather than the motion of individual charges. To describe the magnetic field due to a current, we have the *Biot-Savart law*:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{s} \times \vec{r}}{r^3}$$

Consider the circuit of the figure below. The curved segments are arcs of circles of radii a and b. The straight segments are along the radii. Find the magnetic field at P, assuming a current i in the circuit. (15%)



3. (a) We can write the electric and magnetic fields in the usual mathematical form of a sinusoidal traveling wave:

$$E(x,t) = E_m \sin(kx - \omega t), \qquad B(x,t) = B_m \sin(kx - \omega t)$$

From Faraday' s law, we have

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

Find the ratio of the amplitudes of the electric to the magnetic components of the wave. (b) A vector called the *Poynting vector* is defined by

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \ .$$

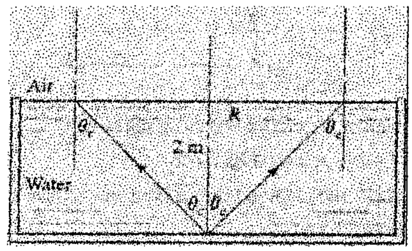
Please explain what this vector means. Note that it is a vector. (c) A radio station radiates a sinusoidal wave with an average total power of 50 kW. Assuming it radiates equally in all directions (which is unlikely in real-world situations), find the amplitudes of the electric and the magnetic components of the wave at a distance of 100 km from the antenna. (2%+4%+9%)

4.(a) Find the speed and the kinetic energy of a neutron having a de Broglie wavelength of 0.1 nm, typical of atomic spacing in crystals. (b) Label as true or false these statements involving the quantum numbers n, l, m_l. If a statement is false, write the correct one. (1) The values of m_l that are allowed depend only on l and not on n. (2) The n=4 shell contains four subshells. (3) The smallest value of n that can go with a given l is l+1. (4) All states with l=0 also have m_l=0, regardless of the value of n. (5) Every shell contains n subshells. (6) One of these subshells cannot exist: n=2, l=1; n=4, l=3; n=3, l=2; n=1, l=1. (c) For a hydrogen atom with l=1, the measured value of L_z is 0. Find the possible values L_x can have. (6%+2%×6+2%)

5. Consider a grating with N slits, where the slit width a is assumed to be much smaller than the light wavelength. (a) Derive the intensity expression for the diffracted lights on the screen that is assumed far way from the grating. (b) Find the positions of the principal maxima of the diffraction pattern. (15%+5%)

6. You are required to solve one of the following two problems. Surely you could solve both, but no extra marks will be given.

(1). (a) As shown in the figure below, at the bottom of a pool with water (n=4/3) 2.00 m deep, light rays emit upward in all directions. A circular area of light is formed at the surface of the water. Determine the radius R of the light circle. (b) Show that the light rays close to the normal appear to come from a point (2.00 /n=1.5) m below the surface. (c) Suppose there is a camera that can take photos in the water. Now it is exactly beside the luminous body in question *(a)* at the bottom of the pool. A big mirror, whose surface parallel with the surface of the pool water, is directly over the pool surface by 4 m. The object of the camera is to take a photo of the image of the luminous boby formed by the mirror. Find how far away it will appear to the camera. To solve this, you can use the result of question *(b)*. (6%+5%+4%)



(2). By making clever use of the *correspondence principle*, Bohr calculated that the energies of the orbital quantum states of a hydrogen atom are given by

$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2},$$

where m is the electron mass, e is the elementary charge, h is the Plank constant and n is a quantum number. (a) A neutral helium atom has two electrons. Suppose one of them is removed, leaving the helium ion He^+. This ion, with its single electron, resembles the hydrogen atom, except that its nuclear charge is $+2e$ rather than $+e$. Find the wavelength of the photon emitted when the electron makes a transition from the orbit $n=5$ to the orbit $n=2$. (b) The shell with n =1 is called the K shell, that with n=2 the L shell, that with n=3 the M shell and that with n=4 the N shell. If an electron falls from the L shell and moves in to fill a hole in the K shell, we have the K_α line; if it falls from the M shell, we have K_β line. If an electron falls from the M shell to a hole in the L shell, we have the L_α line; if it falls from the N shell, we have L_β line. Let λ_α and λ_β represent the wavelengths of the K_α and K_β x-rays of an element. Show that the Bohr theory gives

$$\frac{\lambda_\beta}{\lambda_\alpha} = k\frac{(Z-1)^2}{Z^2-9}$$

and find the numerical value of k. (c) Find the similar result for the relation between L_α and L_β x-rays. (6%+5%+4%)

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2006～2007 学年第二学期期末考试试卷参考答案

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1 There are six interaction terms , one for every charge pair, number the charges clockwise from the upper left hand corner, then

$u_{12} = q^2 / 4\pi\varepsilon_0\alpha$ ……2'

$u_{23} = q^2 / 4\pi\varepsilon_0\alpha$ ……2'

$u_{34} = q^2 / 4\pi\varepsilon_0\alpha$ ……2'

$u_{41} = q^2 / 4\pi\varepsilon_0\alpha$ ……2'

$u_{13} = q^2 / 4\pi\varepsilon_0(\sqrt{2}\alpha)$ ……2.5'

$u_{24} = q^2 / 4\pi\varepsilon_0(\sqrt{2}\alpha)$ ……2.5'

add these terms and get $u = \dfrac{q^2}{4\pi\varepsilon_0\alpha}(4+\sqrt{2})$ ……1'

the amount of work required is W=u ……2'

2 There are four current segments that could contribute to the magnetic field . the straight segments , however , contribute nothing because the straight segments carry currents either directly toward or directly away from the point P that leaves the two rounded segments. Each contribution to $\vec{B}$ can be found by :

$$B = \int \frac{\mu_0 i d\vec{s} \times \vec{r}}{4\pi r^3}$$

$$= \frac{\mu_0 i}{4\pi} \int_0^\theta \frac{r d\theta}{r^2}$$

$$= \frac{\mu_0 i\theta}{4\pi r} \text{ ……6'}$$

The contribution to $\vec{B}$ from the top arc is : $\dfrac{\mu_0 i\theta}{4\pi a}$ ……2'

The direction is into the page .

There is also a contribution from the above arc : $\frac{\mu_0 i\theta}{4\pi b}$ ....2' the direction is out of the page .

The net magnetic field at P is then : $B = B_1 + B_2 = \frac{\mu_0 i\theta}{4\pi a} - \frac{\mu_0 i\theta}{4\pi b}$ ....2'

The direction is out of the page . ...1'

3. (a)

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$E_m \frac{\partial}{\partial x}\sin(kx - \omega t) = -B_m \frac{\partial}{\partial t}\sin(kx - \omega t)$$

$$E_m k = B_m \omega$$ 1'

$$\frac{E_m}{B_m} = \frac{\omega}{k} = c$$ 1'

(b)

The magnitude of the Poynting vector is the magnitude of the energy flow of the electromagnetic wave, it means the electromagnetic power per unit area in the space. 3'

The direction of the vector means where the energy of the wave is propogating. 1'

It is also a function of the time, which means it varies with the time. We more often than not pay attention to its average value.

(c).

First We consider the magnitude of the Poynting vector. We surround the antenna with an imaginary sphere of radius 100km. This sphere has area

$$A = 4\pi R^2 = 12.6 \times 10^{10} m^2$$

All the power radiated passes through this surface , so the power per unit area is

$$S = \frac{P}{A} = 3.98 \times 10^{-7} W \cdot m^{-2}$$ 2'

From the result of (a), one has

$$S_{av} = \frac{E_m B_m}{2\mu_0} = \frac{E_m^2}{2\mu_0 c}$$ 3'

so

$$E_m = \sqrt{2\mu_0 c S_{av}}$$ 1.5'

$$= 1.73 \times 10^{-2} N/C$$ 0.5'

$$B_m = E_m / c$$ 1.5'

$$= 5.77 \times 10^{-11} T$$ 0.5'

4 (1) $\upsilon = \frac{h}{\lambda m} = \frac{6.626 * 10^{-34} j.s}{(0.1 * 10^{-9} m)(1.675 * 10^{-27} kg)} = 3.96 * 10^{3} m.s^{-1}$ ....3'

$$K = \frac{1}{2} m\upsilon^2 = \frac{1}{2}(1.675 * 10^{-27} kg)(3.96 * 10^{3} m \cdot s^{-1})^2 = 1.31 * 10^{-29} j = 0.0818 eV$$

.......................3'

(2) All of the six statements are true .... 2'×6 .

(3) $L_x = \frac{h}{2\pi}$ ....1'

$L_x = -\frac{h}{2\pi}$ ....1'

5. (a). For every slit, the magnitude of the phasor is

$$E_i = E_0 e^{i(kr_i - \omega t)} \qquad \text{where i=1,2,3,....}$$ 4'

Then the total magnitude is

$$E = \sum_i E_i = E_0 e^{i\omega t} \sum e^{ikr_i}$$

$$= E_0 e^{i(kr_i - \omega t)} \left[1 + e^{ikd \sin\theta} + e^{2ikd \sin\theta} + e^{3ikd \sin\theta} + \cdots + e^{ikd(N-1)\sin\theta}\right]$$

$$= E_0 e^{i(kr_i - \omega t)} \frac{e^{ikdN \sin\theta} - 1}{e^{ikd \sin\theta} - 1}$$ 5'

$$= E_0 e^{i(kr_i - \omega t)} \frac{e^{i\frac{kdN \sin\theta}{2}}}{e^{i\frac{kd \sin\theta}{2}}} \cdot \frac{e^{i\frac{kdN \sin\theta}{2}} - e^{-i\frac{kdN \sin\theta}{2}}}{e^{i\frac{kd \sin\theta}{2}} - e^{-i\frac{kd \sin\theta}{2}}}$$ 3'

So the intensity reads

$$I = I_m \left| e^{i(kr_i - \omega t)} \frac{e^{i\frac{kdN \sin\theta}{2}}}{e^{i\frac{kd \sin\theta}{2}}} \cdot \frac{e^{i\frac{kdN \sin\theta}{2}} - e^{-i\frac{kdN \sin\theta}{2}}}{e^{i\frac{kd \sin\theta}{2}} - e^{-i\frac{kd \sin\theta}{2}}} \right|^2$$

$$= I_m \left(\frac{\sin \frac{kdN \sin\theta}{2}}{\sin \frac{kd \sin\theta}{2}} \right)^2$$ 3'

(b). A principal maximam lies around the center where

$$\sin\frac{kd\sin\theta}{2}=0 \quad \text{and hence} \quad \sin\frac{kdN\sin\theta}{2}=0 \qquad 4'$$

which gives

$$d\sin\theta = m\lambda \qquad m=0,\pm1,\pm2,\ldots \qquad 1'$$

6.

(1). (a).The circular area is formed by rays refracted into the air. The angle θ_c must be the critical angle, because total internal reflection, and hence no refraction, occurs when the angle of incidence in the water is greater than the critical angle. We have, then

$$\sin\ \theta_c = \frac{1}{4/3} \qquad 2'$$

$$R=(2.00m)\tan\theta_c \approx 2.27m \qquad 4'$$

(b). When the light rays are close to the normal, the incident and refraction angle are both small, which validates the following relation

$$\theta \approx \sin\theta \approx \tan\theta \qquad 2'$$

when we draw a simple figure to show the relation between the incident and refracted ray and the angles, it is easy to find

$$\frac{h'}{h}=\frac{\tan\theta_{initial}}{\tan\theta_{refracted}} \approx \frac{\sin\theta_{initial}}{\sin\theta_{refracted}}=1/n \qquad 2'$$

which gives

$$h'=2.00/n=1.50m \qquad 1'$$

Also, when one assumes that the surface of the pool water is a mirror with

$$r\to+\infty$$

one can again get the correct result by applying the equation

$$\frac{n}{h}+\frac{1}{i}=0$$

full marks should be given.

(c). There are three times the image of the luminous body are formed. The first is when the light rays for the first time go through the surface of the pool water, the second time is the rays will be reflacted by the mirror, after these two courses, the image of the body now seems standing at the distance

$$\left(\frac{2.00}{n}+4\right)+4=9.5m \qquad 1'$$

away from the surface of the water. Now for the second time the light rays go through the water surface, opposite to the course of problem (b), the image will be formed

$$\frac{hc}{l_\alpha} = -\frac{me^4}{2\varepsilon_0^2 h^2}\left(\frac{(Z-9)^2}{3^2} - \frac{(Z-1)^2}{2^2}\right) \quad 1'$$

$$\frac{hc}{l_\beta} = -\frac{me^4}{2\varepsilon_0^2 h^2}\left(\frac{(Z-27)^2}{4^2} - \frac{(Z-9)^2}{2^2}\right) \quad 2'$$

then one gets

$$\frac{l_\beta}{l_\alpha} = \frac{20(Z-9)^2}{9(3Z^2-18Z-405)} \quad 1'$$